

- 1) , , , 0,2.  
 , 400 , :  
) 90 ; 340  
) 300 340 ( ).  
 :  
) .

$$P_n(m) \approx \frac{1}{\sqrt{npq}} \cdot \varphi\left(\frac{m - np}{\sqrt{npq}}\right)$$

:  
 $n = 400$  - ;  
 $p = 0,2$  - ,  
 $q = 1 - p = 1 - 0,2 = 0,8$  - ,  
 $m = 90$  - , .

:  
$$\frac{m - np}{\sqrt{npq}} = \frac{90 - 400 \cdot 0,2}{\sqrt{400 \cdot 0,2 \cdot 0,8}} = \frac{10}{\sqrt{64}} = \frac{10}{8} = 1,25$$

$P_{400}(90) \approx \frac{1}{8} \cdot \varphi(1,25) = 0,125 \cdot 0,1826 \approx 0,0228$  - , 400 -  
90 .  
)

$P_n(m_1 \leq x \leq m_2) \approx \Phi(k_2) - \Phi(k_1)$  ;

:  
 $p = 0,8$  - ,  
 $q = 0,2$  - ,  
 $m_1 = 300, m_2 = 340$

$k_1 \quad k_2$  :

$$k_2 = \frac{340 - 400 \cdot 0,8}{\sqrt{64}} = \frac{20}{8} = 2,5 ;$$

$$k_1 = \frac{300 - 400 \cdot 0,8}{\sqrt{64}} = \frac{-20}{8} = -2,5 .$$

:  
 $P_{400}(300 \leq x \leq 340) \approx \Phi(2,5) - \Phi(-2,5) = \Phi(2,5) + \Phi(2,5) = 2\Phi(2,5) = 2 \cdot 0,4938 = 0,9876$   
- , 400 300 340 .

: )  $P_{400}(90) \approx 0,0228$  )  $P_{400}(300 \leq x \leq 340) \approx 0,9876$

2)

0,2, 0,3 0,6

:  $p_1 = 0,2, p_2 = 0,3, p_3 = 0,6$  -

$q_1 = 1 - p_1 = 1 - 0,2 = 0,8$

$q_2 = 1 - p_2 = 1 - 0,3 = 0,7$

$q_3 = 1 - p_3 = 1 - 0,6 = 0,4$

X -

0)  $X = 0$  ( )

$p(0) = q_1 q_2 q_3 = 0,8 \cdot 0,7 \cdot 0,4 = 0,224$

1)  $X = 1$

$p(1) = p_1 q_2 q_3 + q_1 p_2 q_3 + q_1 q_2 p_3 = 0,2 \cdot 0,7 \cdot 0,4 + 0,8 \cdot 0,3 \cdot 0,4 + 0,8 \cdot 0,7 \cdot 0,6 = 0,056 + 0,096 + 0,336 = 0,488$

2)  $X = 2$

$p(2) = p_1 p_2 q_3 + p_1 q_2 p_3 + q_1 p_2 p_3 = 0,2 \cdot 0,3 \cdot 0,4 + 0,2 \cdot 0,7 \cdot 0,6 + 0,8 \cdot 0,3 \cdot 0,6 = 0,024 + 0,084 + 0,144 = 0,252$

3)  $X = 3$

$p(3) = p_1 p_2 p_3 = 0,2 \cdot 0,3 \cdot 0,6 = 0,036$

$x_i$	0	1	2	3
$p(i)$	0,224	0,488	0,252	0,036

:  $0,224 + 0,488 + 0,252 + 0,036 = 1$

$M(X)$

$D(X)$

:

$x_i$	0	1	2	3	:
$p_i$	0,224	0,488	0,252	0,036	1
$x_i p_i$	0	0,488	0,504	0,108	1,1
$x_i^2 p_i$	0	0,488	1,008	0,324	1,82

$$: M(X) = 1,1$$

:

$$D(X) = M(X^2) - (M(X))^2 = 1,82 - (1,1)^2 = 1,82 - 1,21 = 0,61.$$

:

$x_i$	0	1	2	3
$p(i)$	0,224	0,488	0,252	0,036

$$M(X) = 1,1$$

$$D(X) = 0,61$$

3)

X

:

$$\varphi(x) = \begin{cases} 0, & x < 1 \\ \frac{1}{4}, & 1 \leq x \leq b \\ 0, & x > b \end{cases}$$

:

$$) \quad b;$$

$$[1,5; 4,5].$$

:

:

$$\int_{-\infty}^{+\infty} \varphi(x) dx = 1$$

:

$$\frac{1}{4} \int_1^b dx = 1 \Rightarrow \int_1^b dx = 4$$

$$(x) \Big|_1^b = 4$$

$$b - 1 = 4 \Rightarrow b = 5$$

:

$$\varphi(x) = \begin{cases} 0, & x < 1 \\ \frac{1}{4}, & 1 \leq x \leq 5 \\ 0, & x > 5 \end{cases}$$

$$[1,5; 4,5]$$

$$M(x) = \int_{-\infty}^{+\infty} x\varphi(x)dx = \frac{1}{4} \int_1^5 x dx = \frac{1}{4} \cdot \frac{1}{2} (x^2) \Big|_1^5 = \frac{1}{8} (25 - 1) = \frac{1}{8} \cdot 24 = 3$$

$$D(x) = \int_{-\infty}^{+\infty} x^2\varphi(x)dx - (M(x))^2 = \frac{1}{4} \int_1^5 x^2 dx - 3^2 = \frac{1}{4} \cdot \frac{1}{3} (x^3) \Big|_1^5 - 9 =$$

$$= \frac{1}{12} (125 - 1) - 9 = \frac{1}{12} \cdot 124 - 9 = \frac{31}{3} - 9 = \frac{4}{3}$$

$$P(|X - M(x)| \leq \varepsilon) \geq 1 - \frac{D(x)}{\varepsilon^2}$$

$$: \varepsilon = \frac{3}{2} \quad ([1,5; 4,5])$$

$$P\left(|X - 3| \leq \frac{3}{2}\right) \geq 1 - \frac{\frac{4}{3}}{\left(\frac{3}{2}\right)^2} = 1 - \frac{4}{3} \cdot \frac{4}{9} = 1 - \frac{16}{27} \approx 0,41$$

$$P(1,5 \leq X \leq 4,5) = \int_{1,5}^{4,5} \varphi(x) dx = \frac{1}{4} \int_{1,5}^{4,5} dx = \frac{1}{4} (x) \Big|_{1,5}^{4,5} = \frac{1}{4} (4,5 - 1,5) = \frac{1}{4} \cdot 3 = 0,75$$

$$P(1,5 \leq X \leq 4,5) = 0,75$$

$$P\left(|X - 3| \leq \frac{3}{2}\right) \approx 0,41.$$

$$: b = 5 \quad P\left(|X - 3| \leq \frac{3}{2}\right) \approx 0,41 \quad P(1,5 \leq X \leq 4,5) = 0,75$$

4)

100

	2	2-4	4-6	6-8	8-10	10-12	12	
	6	10	21	39	15	6	3	100

) : , 0,9946 -  
 ) ; -  
 6 10%  
 ( ) ; 0,9907 -  
 , 6 -  
 10% ( -  
 ).

	1	3	5	7	9	11	13	
$x_i$	6	10	21	39	15	6	3	
$f_i$	6	30	105	273	135	66	39	100
$x_i f_i$	6	90	525	1911	1215	726	507	654
$x_i^2 f_i$	1	3	5	7	9	11	13	4980

$$\bar{x} = \frac{\sum x_i f_i}{\sum f_i} = \frac{654}{100} = 6,54$$

$$\sigma^2 = \frac{\sum x_i^2 f_i}{\sum f_i} - (\bar{x})^2 = \frac{4980}{100} - 6,54^2 = 49,8 - 42,7716 = 7,0284.$$

$$s^2 = \frac{f}{f-1} \cdot \sigma^2 = \frac{100}{99} \cdot 7,0284 \approx 7,0994$$

) , 0,9946 -

$$\mu_x = \sqrt{\frac{s^2}{f} \left(1 - \frac{f}{F}\right)}$$

F

$$\frac{f}{F} \rightarrow 0.$$

$$\mu_x = \sqrt{\frac{s^2}{f}} = \sqrt{\frac{7,0994}{100}} \approx 0,2664$$

t

$$2\Phi(t) = \gamma, \quad \Phi(x) -$$

$$\gamma = 0,9946$$

$$: t = 2,78$$

$$\Delta_x = t\mu_x = 2,78 \cdot 0,2664 \approx 0,74$$

$$0,9946$$

$$\begin{aligned} \bar{x} - \Delta_x < \tilde{x} < \bar{x} + \Delta_x \\ 6,54 - 0,74 < \tilde{x} < 6,54 + 0,74 \\ 5,80 < \tilde{x} < 7,28 \end{aligned}$$

)  
10% ( 6 )

$$t = \sqrt{\frac{f \cdot \Delta^2}{\tilde{p}(1-\tilde{p})}}$$

$$6 : \tilde{p} = \frac{6+10+21}{100} = \frac{37}{100} = 0,37$$

$$\Delta = 0,1$$

$$t = \sqrt{\frac{100 \cdot (0,1)^2}{0,37 \cdot 0,63}} \approx 2,07$$

$$\gamma \approx 2\Phi(t) = 2\Phi(2,07) = 2 \cdot 0,4808 = 0,9615$$

) , 0,9907  
10% ( 6 )

$$f = \frac{\tilde{p}(1-\tilde{p})t^2}{\Delta^2}$$

$$2\Phi(t) = 0,9907$$

$$\Phi(t) = 0,49535$$

$$t = 2,6$$

$$f = \frac{\tilde{p}(1-\tilde{p})t^2}{\Delta^2} = \frac{0,37 \cdot 0,63 \cdot (2,6)^2}{(0,1)^2} = 157,5756$$

158

$$) 5,80 < \tilde{x} < 7,28$$

$$) \gamma \approx 0,96$$

) 158

5)  $\frac{50}{X (\%)}$   $Y (\%)$  -  
 : -

X	Y					
	5-9	9-13	13-17	17-21	21-25	
15-21	3	2	1			6
21-27	1	2	3	2		8
27-33		2	7	3		12
33-39		2	5	8		15
39-45			2	2	1	5
45-51				2	2	4
	4	8	18	17	3	50

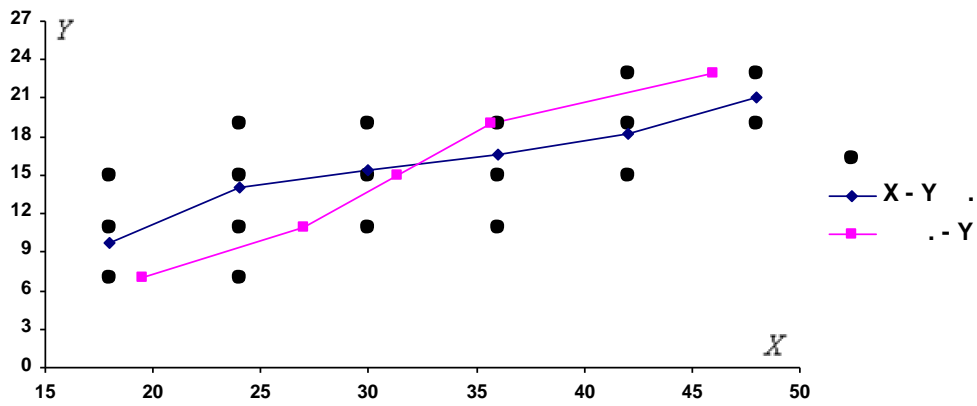
1)  $\bar{x}_i, \bar{y}_j$  -  
 ;  
 2)  $X, Y$  -  
 )  
 ) ;  
 )  $\alpha = 0,05$  X  
 Y;  
 )  
 43%.

1) : , -  
 :

X	Y						$m_j$	$\bar{y}_j$
	7	11	15	19	23			
18	3	2	1			6	9,67	
24	1	2	3	2		8	14	
30		2	7	3		12	15,33	
36		2	5	8		15	16,6	
42			2	2	1	5	18,2	
48				2	2	4	21	
$n_i$	4	8	18	17	3	50		
$\bar{x}_i$	19,5	27	31,33	35,65	46			

$\bar{x}_i \quad \bar{y}_j$  -

(  $(x_i; \bar{y}_j), (\bar{x}_i; y_j)$  ):



2) , X Y  
 :  
 ) ,

X :

$x_i$	18	24	30	36	42	48	:
$n_i$	6	8	12	15	5	4	50
$x_i n_i$	108	192	360	540	210	192	1602
$x_i^2$	324	576	900	1296	1764	2304	
$x_i^2 n_i$	1944	4608	10800	19440	8820	9216	54828

$\bar{x}$ ,  $\sigma_x^2$ ,  $\sigma_x$  :

$$\bar{x} = \frac{\sum x_i n_i}{n} = \frac{1602}{50} = 32,04$$

$$\sigma_x^2 = \frac{\sum x_i^2 n_i}{n} - \bar{x}^2 = \frac{54828}{50} - (32,04)^2 = 1096,56 - 1026,562 \approx 70$$

$$\sigma_x = \sqrt{70} \approx 8,37.$$

Y :

$y_j$	7	11	15	19	23	:
$m_j$	4	8	18	17	3	50
$y_j m_j$	28	88	270	323	69	778
$y_j^2$	49	121	225	361	529	
$y_j^2 m_j$	196	968	4050	6137	1587	12938



$$\bar{y}, \sigma_y^2, \sigma_y :$$

$$\bar{y} = \frac{\sum y_j m_j}{n} = \frac{778}{50} = 15,56$$

$$\sigma_y^2 = \frac{\sum y_j^2 m_j}{n} - \bar{y}^2 = \frac{12938}{50} - (15,56)^2 = 258,76 - 242,114 = 16,65$$

$$\sigma_y = \sqrt{16,65} \approx 4,08$$

X Y -

:

X	Y				
	7	11	15	19	23
18	378	396	270		
24	168	528	1080	912	
30		660	3150	1710	
36		792	2700	5472	
42			1260	1596	966
48				1824	2208

$$\overline{xy} = \frac{\sum \sum x_i y_j n m_{ij}}{n} = \frac{26070}{50} = 521,4.$$

:

$$r_{xy} = \frac{\overline{xy} - \bar{x} \cdot \bar{y}}{\sigma_x \sigma_y} = \frac{521,4 - 32,04 \cdot 15,56}{8,37 \cdot 4,08} \approx 0,67.$$

Y X :

$$y - \bar{y} = r_{xy} \frac{\sigma_y}{\sigma_x} (x - \bar{x})$$

:

$$y - 15,56 = 0,33 \cdot (x - 32,04),$$

$$y = 0,33x + 5,1 -$$

0,33% 1% ( ) ( )

X Y :

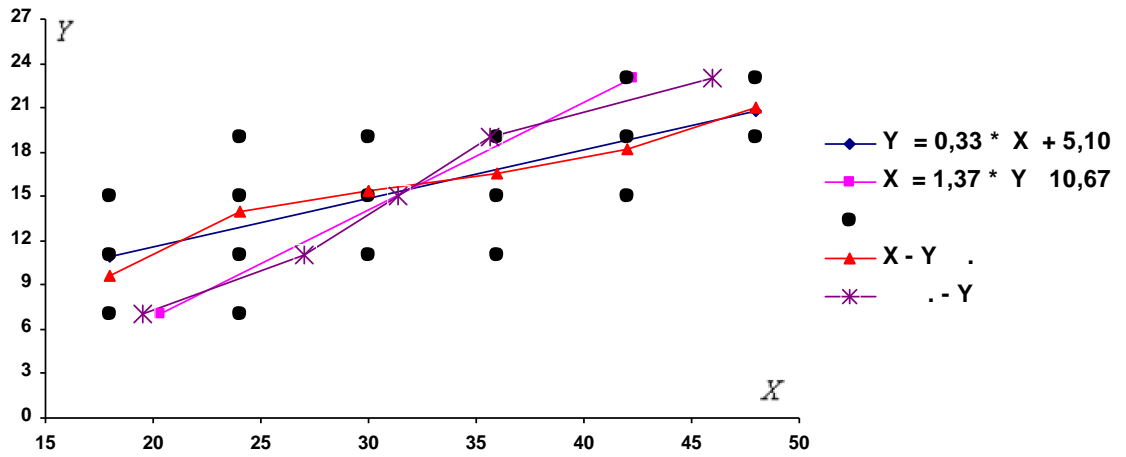
$$x - \bar{x} = r_{xy} \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

:

$$x - 32,04 = 1,37 \cdot (y - 15,56),$$

$$x = 1,37y + 10,67 -$$

1% 1,37% ( ) ( )



)  $r_{xy} \approx 0,67$ .

$\alpha = 0,05$

$H_0$   
 $H_1$

$$t = \frac{r_{xy}}{m_{r_{xy}}}$$

$$m_{r_{xy}} = \sqrt{\frac{1 - r_{xy}^2}{n - 2}} = \sqrt{\frac{1 - 0,67^2}{48}} \approx 0,1072$$

$$t = \frac{0,67}{0,1072} = 6,25$$

$\alpha = 0,05$   
 $t = 2,01$

$f = n - 2 = 48$

$t > t$

43%

$y(43) = 0,33 \cdot 43 + 5,1 = 19,29 \%$